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NOTE ON PROBLEM 118.

BY GEORGE R. DEAN, ROLLA, MO.

The analysis of a geometrical problem has always appeared to me more interesting and useful than the mere construction and proof.

Any problem in maxima and minima may be solved by applying the general principle that a maximum or a minimum value of a variable quantity lies between two equal values. Let P be any point in the line FH . At some other point Q the angle DQC is equal to the angle DPC . The point Q is obviously found by drawing a circle through D , C , and P . The point at which the angle is a maximum is situated between P and Q . It is obvious that if the point P had been so selected that Q would coincide with it, the required point would be determined. A circle drawn through D and C tangent to the given line will cut the line in two coincident points between which the maximum point lies, and is therefore coincident with them.

As an example of this kind of analysis I propose the following problem :

Find a point in a given line such that the sum of its distances from two fixed points is a minimum. Give the analysis.

CALCULUS.

94. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa.

Find the minimum isosceles triangle that can be described about a given ellipse, having its base parallel to the major axis.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.; M. C. STEVENS, A. M., Professor of Higher Mathematics, Purdue University, Lafayette, Ind.; and H. C. WHITAKER, M. E., Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Let x, y be the coördinates of P . Then $x^2/a^2 + y^2/b^2 = 1$ is the equation to the ellipse.

$$CD = b^2/y. \quad \therefore GD = b^2/y + b = b(b+y)/y.$$

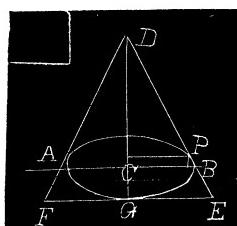
$$AC = a^2/x. \quad \therefore FG = a^2(b+y)/bx.$$

$$\therefore a^2(b+y)^2/xy = \text{area} = \text{minimum}.$$

$$\therefore dy/dx = y(y+b)/x(y-b).$$

Also $dy/dx = -b^2x/a^2y$, from the equation to the ellipse.

$$\therefore \frac{b^2x}{a^2y} = \frac{y(b+y)}{x(b-y)}, \text{ or } x^2 = \frac{a^2y^2(b+y)}{b^2(b-y)}.$$



This value of x^2 in the equation to the ellipses gives, after reduction, $2y^2 + by = b^2$.

$$\therefore y = \frac{1}{2}b \text{ for a minimum.}$$

$$\therefore \text{Altitude} = b(b+y)/y = 3b. \quad \text{Base} = 2a^2(y+b)/bx = 3a^2/x = 2a_1/3.$$

$$\text{Side} = \sqrt{(9b^2 + 3a^2)}. \quad \text{Area} = 3ab_1/3.$$

III. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Let the equation to tangent DE be $(x/a)\cos\phi + (y/b)\sin\phi = 1$.

Then $CB = a\sec\phi$; $CD = b\cosec\phi$. $DG = b + b\cosec\phi$.

Area $= DG \cdot GE$. $GE : CB :: DG : DC$. Whence

$$GE = \frac{a\sec\phi(1 + \cosec\phi)}{\cosec\phi} = \frac{a\sec\phi(1 + \cosec\phi)}{\cosec\phi} b(1 + \cosec\phi) = ab\tan\phi(1 + \cosec\phi)^2$$

Hence $\tan\phi(1 + \cosec\phi)^2$ is to be examined for a minimum. Differentiating and equating to zero, we have

$$\sec^2\phi(1 + \cosec\phi)^2 - 2\tan\phi(1 + \cosec\phi)\cosec\phi\cot\phi = 0,$$

whence, $\sec^2\phi(1 + \cosec\phi) = 2\cosec\phi$, or

$$\frac{1 + \sin\phi}{\sec\phi\cos^2\phi} = \frac{2}{\sin\phi}.$$

Solving, $\sin\phi = \frac{1}{2}$; whence $\cosec\phi = 2$.

Hence $DG = b + b\cosec\phi = b + 2b = 3b$.

Also solved by P. S. BERG, W. H. DRANE, J. SCHEFFER, and J. W. YOUNG.

95. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

A ship starts at the equator and sails northeast at all times. How far has the ship sailed (in miles) when her latitude is $30^\circ, 45^\circ, 60^\circ, 90^\circ$? How far when her longitude is $90^\circ, 180^\circ, 270^\circ, 360^\circ$? Regarding the earth as a sphere, radius 3956 miles.

Solution by the PROPOSER.

Let A be the point of the ship's departure, $APQR$ the ship's course, P, Q two consecutive points on the course, $AG = \theta$ = longitude of P , $PG = \phi$ = latitude of P , $OP = r$ = radius of the earth, $\angle PQN = \beta = \frac{1}{4}\pi$.

Then $PQ = ds$, $PN = EP \times \angle PEN = r\cos\phi d\theta$, $QN = rd\phi$.

$$\therefore ds^2 = r^2(\cos^2\phi d\theta^2 + d\phi^2).$$

$$\therefore ds = r(\cos^2\phi d\theta^2 + d\phi^2)^{\frac{1}{2}} \dots\dots (1).$$

$$PN/QN = \tan\beta = \cos\phi d\theta/d\phi.$$

$$\therefore d\theta = \tan\beta d\phi/\cos\phi \dots\dots (2).$$

$$(2) \text{ in } (1) \text{ gives } ds = \sqrt{(1 + \tan^2\beta)d\phi} = rd\phi/\cos\beta.$$

$$\therefore s = (r/\cos\beta) \int_0^{\phi} d\phi = r\phi/\cos\beta = r\phi\sqrt{2}.$$

When $\phi = \frac{1}{4}\pi$, $s = \frac{1}{4}\pi r\sqrt{2} = 2929.3817$ miles.

When $\phi = \frac{1}{2}\pi$, $s = \frac{1}{2}\pi r\sqrt{2} = 4394.07257$ miles.

When $\phi = \frac{3}{4}\pi$, $s = \frac{3}{4}\pi r\sqrt{2} = 5858.7634$ miles.

When $\phi = \frac{1}{4}\pi$, $s = \frac{1}{4}\pi r\sqrt{2} = 8788.14514$ miles.

